



On Extended Exponential Distribution: Properties and Applications In Tracking the Pandemic Covid-19

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Abstract

This article reviews and discusses an extended form of exponential model distributions as well as some important cases of these distributions. A new four parameters extended exponential model is proposed. The family of the exponential distributions and their derivate models finds many applications. The author proposes a new four parameter extended exponential model distribution, which generalizes the model of Weibull-exponential distributions. The applicability of the new models is well confirmed using two real data sets. Various properties of this new model are investigated and then exploited to derive several related results, especially characterizations in probability. As a motivation, the statistical applications of the results based on health related data are included to investigate the reliability properties of a flexible extended exponential model of distributions. These findings will be useful for the practitioners in various fields of theoretical and applied sciences. We use the general formula to track the spread of the Corona virus in the world for a period of six months in all countries of the world, guess the values of the parameters that reflect the values of indicators that express the expectation of the epidemic and its acceleration in the world, and find margins that can help lead the crisis and manage the pandemic.

Keywords: Exponential; Distribution; Reliability; Quintile; Function; Application; Mixtures; Extended Covid-19

History and Definition

The exponential distribution model was first defined in applied cases as a special case of distribution is one of the widely used continuous distributions. It is a special case of type III Pearson distribution. It is also used for products with constant failure or arrival rates. After a period of 70 years the Exponential Distribution becomes just a special case of the Weibull gamma distribution to appear on its own [1]. Karl Pearson discussed the Weibull and Gamma distributions in 1895. History of exponential density function has been obtained. Generalized exponential density function where it is assumed that independent events occur at a constant rate. Exponential distribution refers to a statistical distribution used to model the time between independent events that happen at the average with constant rate. It is often concerned with the amount of time until some specific event occurs. The Exponential distribution is a continuous

distribution bounded on the lower side. Its shape is always the same, starting at a finite value at the minimum and continuously decreasing at larger x . The exponential distribution decreases rapidly for increasing x . The Exponentiated exponential or generalized exponential distribution is introduced as a special case of the exponentiated Weibull with three parameters. Exponentiated exponential distribution has many mathematical properties which have not been known completely or have not been known in simple and general forms. This distribution presented a comprehensive survey of the mathematical properties and most attractive generalization of the exponential distribution, and has received widespread attention [2]. The generalized exponential (GE) distribution with three-parameter (scale, shape, and location) proved its suitability in probabilistic models related to Rayleigh distribution or recurrence modelling [3]. The GE distribution shares many physical properties of the gamma and

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Weibull distributions, unlike the two parameter exponential distribution preserves its memory property strength [2-6].

The One-parameter Exponential Distribution Model Definition

Assume X be a random variable, the probability density function (pdf) with a parameter can be defined using an alternative parameterization. It describes the arrival time of a randomly recurring independent event sequence. If is the mean waiting time for the next event recurrence, its pdf is given by [7]:

$$f_X(x, \lambda) = \begin{cases} \frac{1}{\lambda} \exp\left\{-\frac{1}{\lambda}x\right\} & x > 0 \\ 0 & x \leq 0 \end{cases} \quad (1)$$

Where represent the scale parameter and is called the rate of the distribution is characterized by the single parameter, or alternatively the most commonly used form of the Exponential

distribution. If be a positive real number we write $X \sim$ exponential (λ) and say that X is an exponential random variable with parameter λ if the pdf is:

$$f_X(x, \lambda) = \begin{cases} \lambda \exp\{-\lambda x\} & x > 0 \\ 0 & x \leq 0 \end{cases} \quad (1)'$$

The basic parameter expresses the waiting time in random processes with exponential distribution, X, before an event occurs will have an exponential distribution if the probability of the event to occur during a certain time interval is directly proportional to the length of that time interval. Exponential distribution is a special case of the two parameter gamma distribution. The representation of the shape of the pdf for exponential distribution with single parameter is shown in figures (Figure 1).

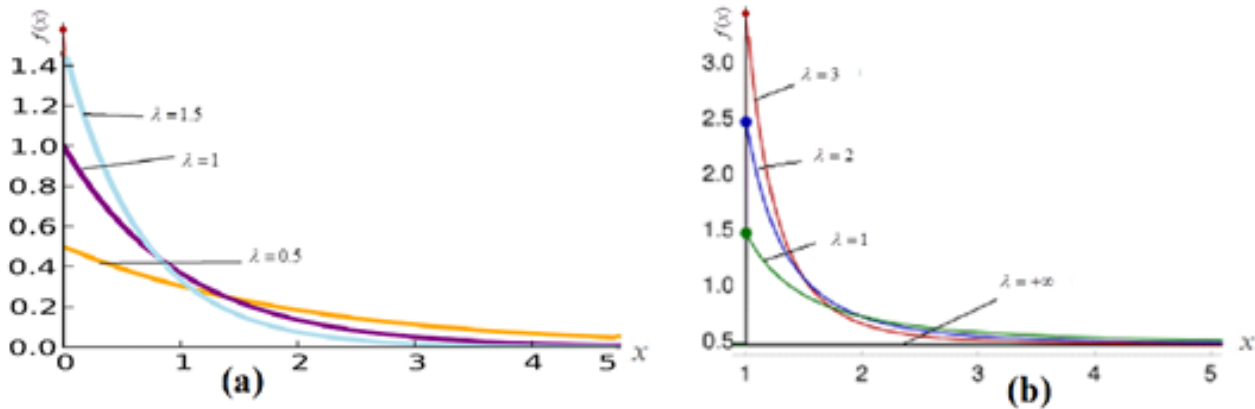
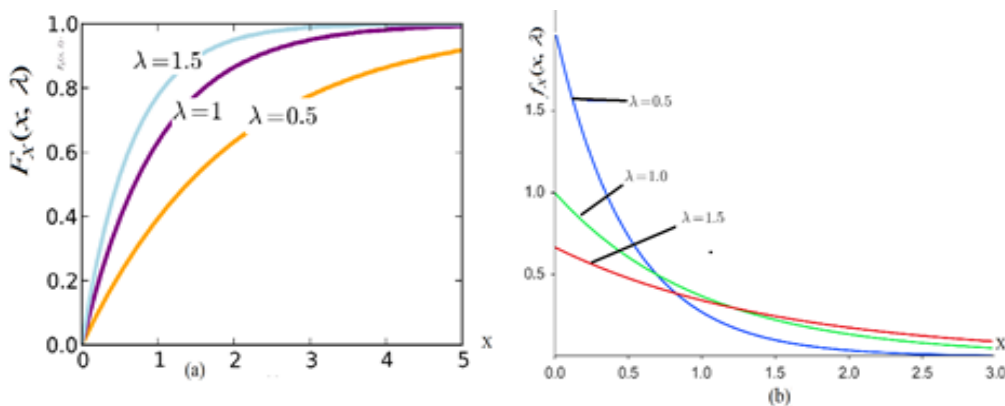


Figure 1: Graph of pdf for various values of λ .



Figures 2: Graphs of pdf and CDF for various values of λ .

The probability distribution function (CDF) can be formulated when integration takes, and it becomes CDF of the exponential random variable with parameter is given by:

$$F_X(x, \lambda) = \begin{cases} 1 - \exp\{-\lambda x\} & x > 0 \\ 0 & x \leq 0 \end{cases} \quad (1)''$$

In order to clarify the basic conditions of this function we show some special shapes for exponential its probability density function and cumulative distribution function for various values of the parameter is shown in figures (Figure 2).

Properties of the One-parameter Exponential Distribution Model

The following properties shows the important properties of the exponential distribution.

Property 1: The exponential distribution has the scaling property which means the random variable follows an exponential distribution with the mean $\lambda > 0$, then for any constant $k > 0$, kX is also an exponential random variable.

Proof: The results are determined using the transformation technique. The proof is straightforward.

Property 2: If $(X_i)_{i=1}^n$ are independent random variables where each follows an exponential distribution with parameter λ_i . For $i = 1, \dots, n$ then $\text{Min}(X_1, \dots, X_i, \dots, X_n)$ is exponential with parameter $\sum_{i=1}^n \lambda_i$. This completes the proof.

Proof: The result follows (Minimum of exponentials) by using the transformation technique such that:

$$P(\text{Min}(X_1, X_2, X_3, \dots, X_n) > x) = \Pr(X_1 > x, X_2 > x, X_3 > x, \dots, X_n > x) \\ = \prod_{i=1}^n \Pr(X_i > x) = \prod_{i=1}^n \exp(-\lambda_i x) = \exp\left(-\left(\sum_{i=1}^n \lambda_i\right)x\right)$$

This completes the proof.

Result 1: If $(X_i)_{i=1}^n$ are independent random variables where each X_i follows an exponential distribution with parameter λ .

For $i = 1, \dots, n$ then $\text{Min}(X_1, \dots, X_i, \dots, X_n)$ is exponential with parameter $n\lambda$.

Proof: The result follows

$$P(\text{Min}(X_1, X_2, X_3, \dots, X_n) > x) = \Pr(X_i > x) \quad ; i = 1, \dots, n \\ = \prod_{i=1}^n \Pr(X_i > x) = \prod_{i=1}^n \exp(-\lambda x) = \exp\left(-x \sum_{i=1}^n \lambda\right) = \exp(-nx\lambda)$$

This concludes the proof.

Result 2: If X is an exponential random variable with parameter λ , then:

- $1 + X$ is a Benktander-Weibull random variable with parameter $(\lambda, 1)$.
- $\text{Exp}(X)$ a random variable follows Beta distribution with parameter $(\lambda, 1)$.
- \sqrt{X} is a Rayleigh random variable with parameter $(1/\sqrt{2\lambda})$.
- $\lim_{n \rightarrow +\infty} \{n \text{Beta}(n, 1)\} = \exp(1)$

Proof: The Formulae and equations expressed by results are produced using basic transformations.

Property 3: Let X be random variable with exponential distribution. Then

- $E(X) = \lambda^{-1}$.
- $V(X) = \lambda^{-2}$.

Proof: So, if $X \sim \text{exponential}(\lambda)$, then

$$E(X) = \int_0^{+\infty} x(\lambda e^{-\lambda x}) dx = \lim_{t \rightarrow +\infty} \int_0^t x(\lambda e^{-\lambda x}) dx$$

Let $x = u$ and $dv = (\lambda e^{-\lambda x}) dx$. Then $dx = du$ and $v = (e^{-\lambda x}) dx$. So

$$E(X) = \lim_{t \rightarrow +\infty} \left([uv]_0^t - \int_0^t v d \right) = \lim_{t \rightarrow +\infty} \left([xe^{-\lambda x}]_0^t - \int_0^t e^{-\lambda x} dx \right) \\ = \lim_{t \rightarrow +\infty} \left(te^{-\lambda t} - 0 - \frac{1}{\lambda} e^{-\lambda t} + \frac{1}{\lambda} \right) = \frac{1}{\lambda}$$

By a similar computation $V(X) = \lambda^{-2}$.

This completes the proof.

Property 4: Let X be random variable with exponential distribution, then the coefficient of variation and moment generating function are:

$$\frac{E(X)}{\sqrt{V(X)}} = 1.$$

$$\Phi(t) = E(e^{tX}) = \lambda(\lambda - t)^{-1}.$$

Proof: The result is obtained using the transformation techniques.

Property 5: Let X and Y are independent exponential random variables with parameter λ . Then $X + Y$ and X / Y are independent.

Proof: Let X and Y are independent exponential random variables with parameter λ . Then the joint density function of X / Y is $f_{XY}(x, y) = \lambda^2 e^{-\lambda(x+y)}$ because X, Y are independent

$$\begin{cases} W = X + Y \\ V = \frac{X}{Y} \end{cases} \Rightarrow \begin{cases} X = \frac{WV}{1+V} \\ Y = \frac{W}{1+V} \end{cases}$$

$$|J| = \left| \begin{array}{cc} \frac{1}{1+v} & \frac{-w}{(1+v)^2} \\ \frac{v}{1+v} & \frac{w}{(1+v)^2} \end{array} \right| = \frac{w}{(1+v)^2}$$

Then the Jacobean

$$\text{And } f_{WV}(w, v) = \lambda^2 e^{-\lambda w} \cdot \frac{w}{(1+v)^2} = (\lambda^2 e^{-\lambda w} \cdot w) \cdot \frac{1}{(1+v)^2}$$

this prove $X + Y, X/Y$ are independent.

Result 1: Assume X and Y be independent random variables where each X and Y follow an exponential distribution with parameter $\lambda = 1$, then X/Y follows a Fisher distribution ($F_{(2,2)}$).

Proof: The random variables X and Y are independent have exponential distribution respectively, $f(x) = e^{-x}, f(y) = e^{-y}$ and $f(x, y) = e^{-(x+y)}$. Let $U = X$ and $V = X/Y$, then the

Jacobian $|J| = -\frac{U}{V^2}$ and

$$f_{UV}(u, v) = f_{XY}(u, \frac{u}{v}) \left| \frac{-U}{V^2} \right| = e^{-u(1+\frac{1}{v})} \left(\frac{u}{v^2} \right) \text{ for simple}$$

transformation we take the probability density function:

$$f_V(v) = \int_0^\infty e^{-u(1+1/v)} \left(\frac{u}{v^2} \right) du = \frac{1}{v^2} \int_1^\infty e^{-u(1+1/v)} u \frac{(1+1/v)^2}{\Gamma(2)} du \frac{\Gamma(2)}{(1+1/v)^2} = \frac{1}{(1+1/v)}$$

This is exactly the pdf of $F_{(2,2)}$, which completes the proof.

Property 6: The exponential distribution with no memory.

Proof: Let X be Random variable with exponential distribution with parameter λ .

$$\begin{aligned} P(X > t + t_0 / t_0) &= \frac{P(X > t + t_0) \cap P(X > t_0)}{P(X > t_0)} = \frac{1 - F_X(t + t_0)}{1 - F_X(t_0)} \\ &= \frac{\exp\{-\lambda(t + t_0)\}}{\exp\{-\lambda t_0\}} = \exp\{-\lambda t\} = P(X > t), \forall t_0, t > 0 \end{aligned}$$

This completes the proof.

Recent Modifications in Extended Exponential Distribution

The generalized exponential distribution is not the same as the class of exponential families, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the normal, binomial, Weibull, and many others. During the review of the exponential and

generalized exponential distributions, it is found that these letters can explain a number of physical phenomena which could be proposed as alternative extensions of the extended exponential distribution. The two-parameter exponentiated exponential or the generalized exponential distribution is considered as a special case representing more phenomena distributions. The pdf of an exponential distribution with two parameters can be defined using an alternative parameterization given by:

$$f_X(x, \lambda, \mu) = \begin{cases} \lambda \exp\{-\lambda(x - \mu)\} & x > \mu \\ 0 & x \leq \mu \end{cases} \quad (2)$$

Where λ represents the scale parameter and μ the location parameter. X is a random variable said to have the linear exponential distribution with two parameters λ and μ . The corresponding pdf is given by [18]:

$$f_X(x, \lambda, \mu) = \begin{cases} (\lambda + \mu x) \exp\left\{-\left(\lambda x + \frac{\mu}{2} x^2\right)\right\} & x > \mu \\ 0 & x \leq \mu \end{cases} \quad (2')$$

The result of new distribution, introduced by an interesting method of adding a new parameter to an existing distribution. The pdf of this extended exponential distribution is given by [8]:

$$f_X(x, \alpha, \beta) = \begin{cases} \alpha \beta^{-1} \exp\{-\beta^{-1} x\} [1 - (1 - \alpha) \exp\{-\beta^{-1} x\}]^2, & \alpha, \beta > 0; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (3)$$

The exponential distribution model which was originally introduced by as an extension of the standard one parameter exponential distribution [9]. The ETE distribution results from the mixture of Erlang distribution and the left truncated one-parameter pdf exponential distribution given by:

$$f_X(x, \lambda, \alpha) = \begin{cases} \alpha(1 - e^{-\lambda}) \exp\{-\alpha(1 - e^{-\lambda})x\}, & \lambda, \alpha > 0; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (4)$$

Where α is the shape parameter and λ the scale parameter. The ETE distribution collapses to the classical one-parameter exponential distribution with parameter α when $\lambda \rightarrow +\infty$ [10].

The important most recent statistical literature in modified extensions of the exponential distributions have been proposed to contour such difficulties. For example, an extension of the exponential distribution typically called the generalized exponential distribution. Therefore, it is said that the random variable X follows the general exponential distribution if its pdf is given by [11,12]:

$$f_X(x, \alpha, \beta) = \begin{cases} \alpha \beta \exp\{-\alpha x\} [1 - \exp\{-\alpha x\}]^{\beta-1}, & \alpha, \beta > 0; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (5)$$

Recently, another extension of the exponential model, so that a

random variable X follows the exponential distribution (NHE) if its pdf is given [13,14].

$$f_X(x, \alpha, \beta) = \begin{cases} \alpha\beta(1+\alpha x^2)^{\beta-1} \exp[-(1+\alpha x^2)^\beta], & \alpha, \beta > 0 \quad ; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (6)$$

More recently, other researchers proposed another extension of the exponential distribution call extension exponential distribution [15]. The generalized extended exponential distribution is presented if its pdf is given by [16]:

$$f_X(x, \alpha, \beta) = \begin{cases} \frac{\alpha^2}{(\alpha + \beta)}(1 + \beta x) \exp(-\alpha x), & \alpha, \beta > 0 \quad ; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (7)$$

Moreover, an extended exponential geometric distribution applied for medical data [17]. An extended Lomax distribution was introduced [18]. The Lomax-exponential distribution (LED) is obtained if X follows the exponential distribution with parameters. The corresponding pdf of this new general distribution is given by:

$$f_X(x, \lambda, a, k) = \begin{cases} \frac{\lambda k a^k \exp(\lambda x)}{[(a-1) + \exp(-\alpha x)]^{k+1}}, & \lambda, a, k > 0 \quad ; x > 0 \\ 0; & x \leq 0 \end{cases} \quad (8)$$

It includes the exponential distribution and has a simple relation with the Lomax distribution. When $a = k = 1$, the LED reduces to the two parameter exponential distribution. Then the LED is then considered as a generalized case of the exponential distribution [19]. A new four-parameter continuous model, called the exponentiated Weibull exponential distribution, is introduced based on exponentiated Weibull-G family [20]. The new model Extended Exponential Distribution Model (EEDM) contains some new distributions as well as some former distributions, its pdf is:

$$f_X(x, \alpha, \lambda, a, \beta) = \begin{cases} a\alpha\lambda\beta[e^{2x}-1]^{\beta-1} \exp\left[-\left\{\alpha(e^{2x}-1)^\beta - \lambda x\right\}\left[1 - \exp(-\alpha(e^{2x}-1)^\beta)\right]^{\beta-1}\right], & x > 0 \\ 0; & x \leq 0 \end{cases}$$

Where $\alpha, a, \beta, \lambda > 0$. The new four parameters model extends some recent distributions and provides some new distributions and contains some new distributions as well as some current distributions represent special sub-models of the EWE distribution for examples(Exponentiated Exponential (EEE), new Exponentaited Rayleigh exponential (ERE), new Weibull exponential (WE), Exponential exponential (EE), The Burr X-Exponential distribution has been successfully developed, its statistical properties like the quantile function, median, survival function and its pdf is [21,22]:

$$f_X(x, \theta, \lambda) = \begin{cases} \frac{2\theta\lambda[1-e^{(-\lambda x)}]}{(e^{(-\lambda x)})^2} \exp\left[-\left(\frac{1-e^{(-\lambda x)}}{e^{(-\lambda x)}}\right)^2\right] \left\{1 - \exp\left[-\left(\frac{1-e^{(-\lambda x)}}{e^{(-\lambda x)}}\right)^2\right]\right\}^{\theta-1}, & x > 0 \\ 0; & x \leq 0 \end{cases} \quad (10)$$

For $\theta > 0, \lambda > 0$: where: θ represented a shape parameter and λ a scale parameter. The shape of the Burr X-Exponential distribution could be unimodal in special cases.

The results obtained are in line with existing literature, which shows that the MSE decreases with increased sample size and with the increased value of the extended parameters. The decrease in the MSE also agrees with the submission that increasing information (parameters) of the distribution reduces the uncertainty in the distribution. The results also show that the proposed distribution is more flexible than its other counterparts and will be appropriate for modelling data in dealing with real-life conditions in finance, manufacturing, biological sciences, medical statistics, environmental sciences, etc. Hence, this study has shown that weighting the Extended Exponential Distribution with a new four parameters is more flexible than lower values and that the proposed distribution is applicable in modelling real-life data in medical statistics.

Mixtures of Extended Exponential Distributions

The mixture phenomena models can be quite effective in dealing with such data and for mixture of exponential distributions can be examined analytically by considering the theory of queues in which both the inter arrival time and service-time distributions are mixtures of exponential distributions with specified first two moments, we show that additional information about the distributions is more important for the inter arrival time than for the service time. Arbitrary nonparametric mixtures of exponential are considered as possible models for a lifetime distribution. Exponential mixture model normal mixture distributions have been applied widely in practice to model heterogeneous data, as surveyed, in some circumstances however, the adoption of component densities that are normal is inappropriate, such as in the modelling of failure time. Mixture models arise naturally when one mechanism generates data according to one model and another mechanism generates data according to a different model. Discusses of properties mixed exponential distributions and consider their examples and variations appearing often in insurance [23]. The mixtures of exponential distributions treated as the Laplace transforms of mixing distributions and established some stochastic order relations between them: star order, dispersive order, dilation [24,25]. Mixtures of exponential distributions are always unimodal and skewed, and components with large mean completely overlap the components with small mean. As a result, fitting a mixture of exponential distributions is often a frustrating experience. Consider the simple k-finite exponential mixture model having pdf of the form [12].

$$f_X(x, p, \lambda) = \sum_{j=1}^k p_j \left(\frac{1}{\lambda_j}\right) \exp\left\{-\frac{x}{\lambda_j}\right\}$$

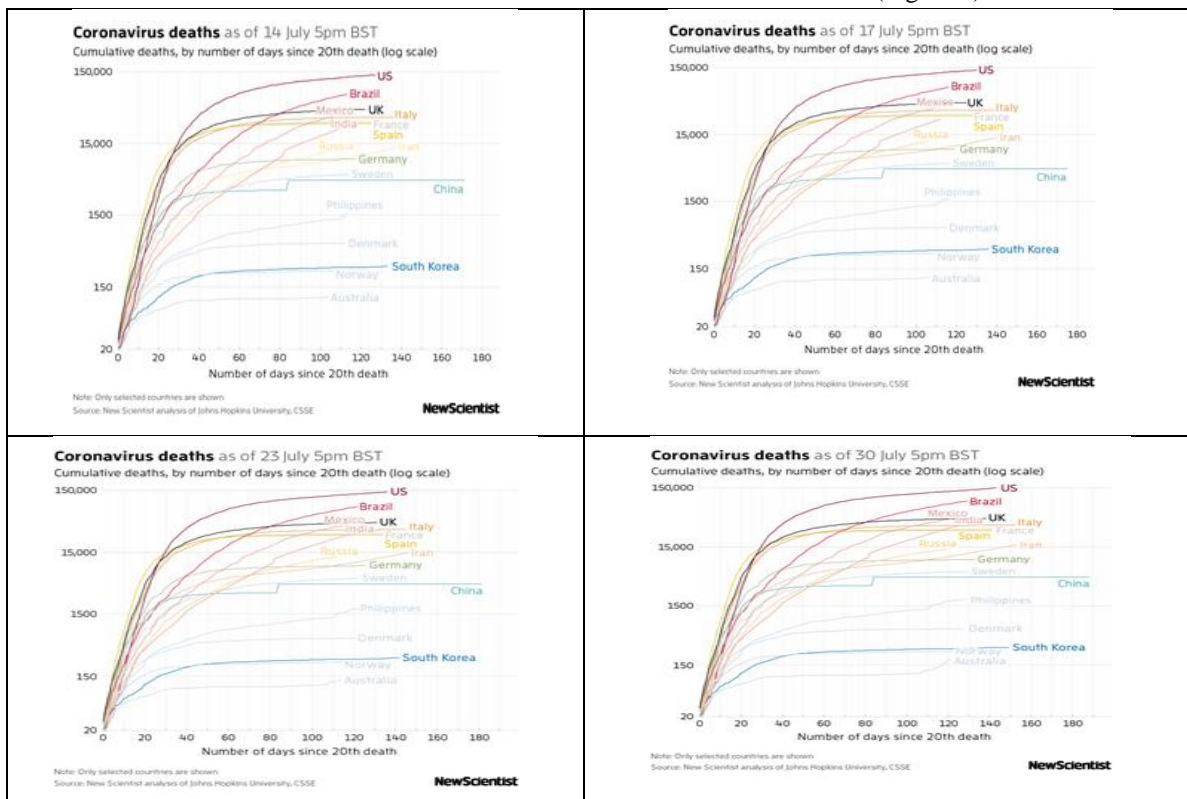
Where $0 \leq p_j \leq 1, j = 1, \dots, k$ for $\sum_{j=1}^k p_j = 1$ and λ_j is the parameter of the exponential distribution of the j^{th} component. They p_j are called mixing proportions and they can be regarded as the probability that a randomly selected observation belongs to the j^{th} subpopulation.

A new class of weighted distributions based on the extended exponential distribution was introduced. The proposed model contains the WE model as its sub model. It is shown that the distribution function, hazard function, and moment generating function can be obtained in closed form. A new class of weighted distributions is proposed by incorporating an extended exponential distribution method is introduced the skew normal distribution and other symmetric distributions have been defined, and several properties and their inference procedures have been discussed by several authors [25-27].

A class of weighted exponential (WE) distribution that has a shape parameter, derived some properties of generalized exponential density function [28]. The exponential distribution is a popular model in applications to real data and exhibit great mathematical tractability [29]. There is a fair amount of variation between the different confidence intervals.

Applications and Conclusions

Property of the exponential distribution with no memory gives the best chance to application to Genetic Distance and describes the time for a continuous process to change state. The rapid increase in the spread of the epidemic follows the exponential model with different parameter values insulating countries and the precautionary measures used in preventing the spread of the krona virus in different areas of spread. Statistics adopted to track the spread of the epidemic indicate that here, it failed to predict the extent of its control, despite the worrying increase in the prevalence and speed. Corona virus spread in Wuhan, China in December 2019, the Corona virus invaded the emerging global transmission in less than three months to almost all countries of the world, and yet there is no medicinal drug that eliminates the virus and prevents its spread. The World Health Organization has declared the virus a global pandemic. About 6 months ago, the new Corona virus invaded the world, infected millions and claimed hundreds of thousands of lives, but with the passage of all this time, what have scientists reached so far regarding knowledge of the path of transmission of this epidemic? Which enables the world to control and lead the crisis before the end of this year? Tracks show the daily number of confirmed cases in four cases of a July month for some countries that have large cases on a large scale over confirmed cases, and they are meaningful when they can be interpreted in light of the extent to which the parameters of the exponential distribution pattern are tested and are shown in (Figure 3).



Figures 3: Graphs of the daily number of confirmed cases from July to some countries.

The worldwide death toll has passed 668,000. The number of confirmed cases is more than 17 million, according to the map and dashboard from Johns Hopkins University, though the true number of cases will be much higher. It is clear from the statistics of the last half of July (14 to 30) that there is an unstable increase in the number of injuries at a high rate and with less proportions with regard to the number of deaths, as shown in the four cases as follows:

14 July worldwide death toll has passed 574,000. The number of confirmed cases is more than 13.1 million, according to the map and dashboard from Johns Hopkins University, though the true number of cases will be much higher. 17 July worldwide death toll has passed 591,000. The number of confirmed cases is more than 13.8 million, according to the map and dashboard from Johns Hopkins University, though the true number of cases will be much higher. 23 July worldwide death toll has passed 624,000. The number of confirmed cases is more than 15.2 million, according to the map and dashboard from Johns Hopkins University, though the true number of cases will be much higher. 30 July worldwide death toll has passed 668,000. The number of confirmed cases is more than 17 million, according to the map and dashboard from Johns Hopkins University, though the true number of cases will be much higher (Figure 4).

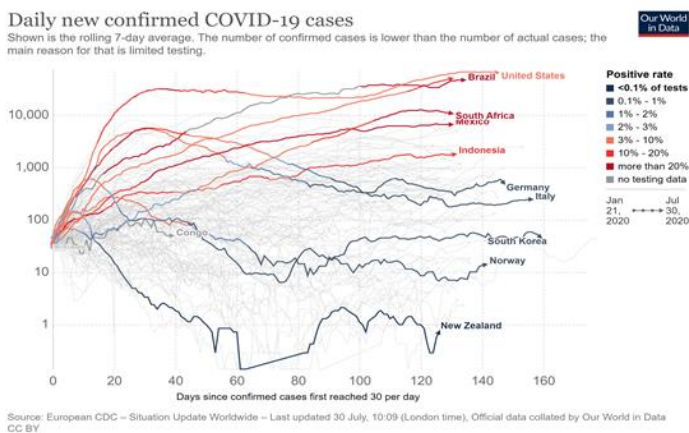


Figure 4: Graph of the stability of confirmed cases in some countries that have adopted safety standards.

Figure 4 shows that the moving average index for 7 days 23-30. The number of confirmed cases is less than. The number of actual cases; the main reason for this is the limited testing and adherence to safety standards adopted in many countries. Figure 4 shows that the exponential distribution indicators are stable beyond the high limits. Some countries have faced the challenge with greater success. Responding successfully means two things: reducing the direct and indirect impact of the epidemic. Perhaps the most important thing to know about a pandemic is that it is possible to combat the

epidemic. The countries that responded most successfully managed to avoid choosing between the two: they avoided the trade-off between a high mortality rate and the higher socio-economic impact of the epidemic. Other countries failed to respond to the epidemic, and these countries suffered a smaller direct impact, but they also restricted the indirect effects because they were able to trigger closures earlier [30].

Conclusion

The study showed that the EEDM fitted was indeed a probability distribution. The parameters of the fitted EEDM like in equation (9) with four parameters (location and the shape parameter) were estimated using appropriate methods; showing all the characteristics of a real distribution function. The distribution was found to approach new exponential as the value increased. In addition, the statistical properties of the fitted EEDM like the expected value, variance, standard error, median; mode, hazard and survival functions, CDF, moments, skewness and kurtosis were obtained. Furthermore, the study recommends further studies using the new model proposed EEDM with four parameters.

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